SUSTAINABLE DEVELOPMENT WITHOUT CONSTRAINTS

by

Lee Endress and James Roumasset

Working Paper No. 00-9
April 2000

SUSTAINABLE DEVELOPMENT WITHOUT CONSTRAINTS

Lee H. Endress* and James Roumasset**

*Asia-Pacific Center for Security Studies
Waikiki Trade Center
2255 Kuhio Avenue, Suite 1900
Honolulu, Hawaii 96815

**Dept. Economics
University of Hawaii
Honolulu 96822

SUSTAINABLE DEVELOPMENT
WITHOUT CONSTRAINTS

Abstract

We explore the possibility of representing sustainability concerns in the objective function of an optimal growth problem instead of as a constraint. In a general model with capital accumulation and resource depletion, we represent intergenerational equity using the pure rate of time preference and the elasticity of the marginal social utility of income and find that a sustainability constraint would be either redundant or render the optimization problem indeterminate. The model also provides a basis for evaluating the depreciation of natural capital for adjusted national income accounts such that maximizing adjusted national income is equivalent to a period-by- period solution of the intertemporal welfare problem. This approach to sustainability rests on the firm theoretical foundations established by three pioneers of economic dynamics and growth: Frank Ramsey, Harold Hotelling, and Tjalling Koopmans.

SUSTAINABLE DEVELOPMENT
WITHOUT CONSTRAINTS

I. INTRODUCTION

Traditional growth theory considers trajectories of output, consumption, and capital accumulation that reflect both dynamic efficiency and maximization of welfare. In attempt to give precision and a neoclassical foundation to the concept of sustainable development, a substantial body of research has arisen that explores sustainability in the context of traditional growth models. Much of this research descends from the Hartwick/Solow result showing that sustaining the value of capital and satisfying efficiency conditions results in a sustainable stream of constant consumption. This is analogous to the case of a renewable resource kept at a steady state level wherein the rate of harvest is sustainable and constant. Subsequent authors have noted that the Hartwick/Solow rule is suboptimal, however, unless the elasticity of the marginal utility of consumption is infinite. This problem has been addressed by adding a constraint to the utilitarian specification of optimal growth, which, in the usual case of a positive utility discount rate, leads to a sustainable and constant consumption level. Unfortunately, the sustainable consumption level may be zero, and what is not survivable can hardly be thought to be optimal.

Several objections can be raised against the imposition of sustainability constraints on growth. Side constraints seem conceptually ad hoc and are not compatible with standard conditions for rational social choice. Moreover, their use generates distorted shadow prices, unnecessarily adding further complication to the accounting of ecological capital in net natural product. As noted by Toman et. al (1995), it would be preferable to represent the sustainability concern as a property of the social welfare function rather than as side constraints. The present paper takes this approach.

The alternative we offer, and the main point of the paper, is that concern for future generations is appropriately captured by a zero pure rate of time preference and that this renders the proposed sustainability constraints redundant. We consider a general model of optimal growth and resource management that addresses earlier objections, advanced on technical or empirical grounds, to a zero rate of time preference (see e.g., Olson and Bailey, 1981 and Heal, 1993). The model incorporates a renewable resource and a backstop technology leading to the possibility of a golden rule steady state. It is the existence of the golden rule steady state for capita accumulation and resource management that permits the social planner to set the rate of time preference equal to zero and still define an optimal path. This approach represents an extension of the Ramsey-Koopmans technique described in Koopmans (1965). The optimal consumption path increases monotonically, asymptotically approximating maximal sustainable consumption. The speed of the approach path is governed by the degree of inequality aversion.

The paper is organized around a basic model of natural resource use, presented in Section II, and its extensions. The model has the advantage of offering considerable generality in that it integrates consideration of non-renewable and renewable resources in a common framework. Section III evaluates maximin and opsustimal welfare in the context of the basic model. In Section IV, we consider ethical and technical issues associated with the rate of time preference and intergenerational equity. Here we take the normative perspective of a social planner and argue the case for neutral weighting across generations. We then derive golden rules of capital accumulation and resource management and present plausible trajectories of per capita consumption leading to the modified golden-rule and golden-rule steady states.

Next, in Section V, we consider net national product (NNP) in the context of the golden rule approach to sustainable development. Using the Ramsey-Koopmans technique, we derive a golden rule net national product and illustrate how NNP evolves along the optimum trajectory. Unlike models with sustainability constraints, shadow prices on produced and natural capital are not distorted. This approach also provides a natural definition of sustainable income such that maximizing sustainable income and maximizing a linear approximation of welfare are equivalent problems. Sustainable income thus defined provides a suitable foundation for full income accounting or sattelite environmental accounts and for incorporating sustainability concerns into the theory of project valuation. Finally, we offer conclusions in Section VI.

In summary, if sustainability is framed as a constraint imposed on the maximization of utilitarian welfare, sustainability is not necessary; the sustainability constraint is likely to be either infeasible or redundant. The notion of sustainability may nonetheless serve to remind us that stewardship for the future involves intergenerational equity as well as dynamic efficiency.

II. AN INTEGRATED MODEL OF NATURAL RESOURCES

The approach to sustainable development pursued in this paper is organized around a general model of natural resource use and its extensions. Consider an economy that uses three inputs, capital (K), labor (L) and a natural resource (R) to produce a single homogeneous good. Assume that the production technology is constant returns to scale, so that the production functions Q(K,R,L) is homogeneous of degree one. For simplicity, we abstract from population growth and technological change and take L = 1. We then set F(K,R) = Q(K,R,L). Following the standard approach, output of production is divided among consumption, gross investment, and the cost of providing the resource as an input to the production process.

Let θ be the unit cost of extracting the natural resource and providing it as an input of production. We assume that this cost is a decreasing function of the resource stock X (see Heal, 1976). Capital depreciation occurs at the rate δK. The basic dynamic equation for this simple economy becomes

$$\dot{K} = F(K,R) - \delta K - \theta(X)R - C \tag{1}$$

The resource stock, X, is drawn down at the rate R. The case of a renewable resource is typically addressed by modeling growth of the resource as a function of the stock level, X.

Representing the growth functions as G(X), the dynamic equation governing the resource stock becomes

$$\dot{X} = G(X) - R \tag{2}$$

For non-renewable resources, G = 0.

We augment this basic model by incorporating a backstop resource. Consider, for example, the case of oil, a non-renewable resource. Oil stocks are drawn down as the economy grows until unit cost, θ, has risen sufficiently to warrant the switch to a superabundant, but high cost, alternative energy source with unit cost θ_b (e.g., coal gasification, solar energy). Once the switch has been made, capital and labor costs alone determine the price of the energy resource: i.e., scarcity rents are no longer significant.

We exploit the convenience of continuous time, and express the social welfare function as

$$W = \int_o^{\infty} U(C_t)e^{-\rho t}\,dt, \qquad (3)$$

where $e^{-\rho t}$ is the utility discount factor and r is the utility discount rate or rate of impatience.

As in Dasgupta and Heal (1979), we assume $\rho \geq 0$, $U^{/}(C) > 0$, $U^{//}(C) < 0$,

$$\lim_{C \to 0} U^{/}(C) = \infty \text{ and } \lim_{C \to \infty} U^{/}(C) = 0.$$

Consistent with these conditions, we will find it useful to employ the iso-elastic utility function

$$U(C) = -C^{-(\eta-1)}, \quad h > 1. \qquad (4)$$

A utilitarian optimum trajectory for consumption and capital accumulation can then be derived as a solution to the following problem, given that such a solution exists:

$$\mathrm{Max} W = \int_o^{\infty} U(C_t)e^{-\rho t}dt \qquad (5)$$

s.t. $\dot{K} = F(K,R) - \delta K - \theta(X)R - C, \quad K(0) = K_0$

$\dot{X} = G(X) - R, \qquad X(0) = X_0$

$\theta < \theta_b$

$X \geq 0$

The Hamiltonian for this problem is

$$H = U(C)e^{-\rho t} + \lambda\,[F(R,K) - \delta K - \theta(X)R - C] + \psi[G(X) - R] \qquad (6)$$

Incorporating the inequality constraints imposed on the problem, we form the Lagrangian

$$L = H + \delta[\theta_b - \theta] + \phi\,\{X\} \qquad (7)$$

The complimentary slackness conditions associated with the inequalities are

$$\delta\,\frac{\partial L}{\partial \delta} = \delta[\theta_b - \theta] = 0 \qquad (8)$$

$$\phi\,\frac{\partial L}{\partial \phi} = \phi X = 0$$

Application of the maximum principle to this optimal control problem yields the following efficiency conditions (see Appendix I):

$$(F_R - \theta) = \frac{1}{(F_K - \delta)}\,\{\dot{F}_R + (F_R - \theta)G'(X) - \theta'(X)G(X)\} \qquad (9)$$

$$\eta\,\frac{\dot{C}}{C} = F_K - (\delta + \rho) \qquad (10)$$

Condition (9) is a generalization of Hotelling's Rule. For the case of a non-renewable resource, G(X) = 0 and equation (9) can be written as

$$\frac{\dot{F}_R}{(F_R - \theta)} = (F_K - \delta) \qquad (11)$$

This is analogous to the familiar "arbitrage" condition,

$$\dot{P} = r\pi\,, \tag{12}$$

from partial equilibrium models of exhaustible resources, where P is the market price of the resource, r is the exogenous interest rate, and p is the producer royalty, given as price minus extraction cost.

For the case of a renewable resource in the steady state where θ_b is non-binding, $\dot{F}_R = 0$ and equation (9) assumes a form similar to that developed in the economics of fisheries (see Clark 1991):

$$(F_K - \delta) = G'(X) - \frac{G(X)\theta'(X)}{(F_R - \theta)} \tag{13}$$

Condition (10) is typically referred to as the Ramsey condition. If a steady state exists, $\dot{C} = 0$ and condition (10) becomes

$$F_K - \delta = \rho. \tag{14}$$

We first use this model to discuss maximin welfare as a special case.

III. MAXIMIN WELFARE AND OPSUSTIMAL GROWTH

Since Rawl's (1971) path-breaking work on the theory of justice, maximin welfare has received considerable attention as an alternative to utilitarianism that provides for intergenerational equity as well as dynamic efficiency. The motivation closest to Rawl's framework is that which appeals to the idea of choice behind the veil of ignorance: "Since no one knows to which generation he or she will belong, the question [of choice] is viewed from the standpoint of each" (Rawls, 1971, p.287). From the so-called 'original position of equal ignorance,' individuals are induced to choose consumption programs that maximize the welfare of the least well-off generation. Dasgupta (1974) sets this notion of welfare in an analytical framework and shows that maximin implies the maximization of constant per-capita consumption over time.

Maximin welfare may be discussed as a special case of our basic model by allowing the parameter η, the elasticity of marginal utility, to approach infinity. A large elasticity implies a high degree of curvature on the utility function, which, in turn, renders a high level of 'egalitarianism' in the distribution of utility across generations. In the limit as $\eta = \infty$, the utilitarian optimum consumption trajectory is flattened. Alternatively, η may be taken as the index of relative risk aversion. The maximin solution would naturally arise if $\eta = \infty$, signifying infinite risk aversion on the part of individuals confronted with a uniform probability distribution of being member of any generation. The result be seen analytically be rearranging equation (11) for the case $\delta = 0$ to give:

$$\frac{\dot{C}}{C} = \frac{F_k - \rho}{\eta} \tag{15}$$

As $\eta \rightarrow \infty$, $\dot{C}/C \rightarrow 0$, generating constant consumption for all $t \geq 0$.

The popularity of the maximin framework for achieving intergenerational equity has not doubt been enhanced by the identification of a simple investment rule, that under certain restrictive conditions is both necessary and sufficient to achieve a maximum level of constant per-capita consumption. That rule, due to Hartwick (1977, 1978), is to follow Hotelling's Rule for resource extraction, equation (9), and invest the profits from the flow of depletion into capital accumulation. More generally, the Hartwick rule may be viewed as keeping the total value of net investment equal to zero.

With consumption C as numeraire, let p and q be the unit prices of capital, K and the natural resource, R, respectively. The Hartwick condition can then be written as

$$p\dot{K} + q\dot{X} = 0 \tag{16}$$

The Hartwick investment rule and the maximin welfare that it renders point to the existence of at least one sustainable consumption path. However, as discussed by Dasgupta and Heal (1979), the maximin welfare criterion leaves countries at the mercy of their initial capital stock. Countries that are capital poor are forever constrained to have lower levels of per capita consumption than more advanced countries that are already capital rich. Also, the assumptions needed to a yield a maximin solution are quite stringent. They include zero population growth in the absence of technological change, no capital depreciation, and output elasticity of capital greater than that of the resource.

These limitations of maximin welfare have led researchers to the concept of opsustimal growth, which has the appearance of a compromise between maximin and utilitarian welfare. The main idea, due to Asheim (1988) and Pezzey (1994), is to apply a non-declining utility constraint to the maximization of utilitarian welfare. Although originally described in the context of non-renewable resources, opsustimal growth could be formulated as an extension of the basic model of Section II through the addition of a side constraint. Specifically, we add the condition $\dot{C} \geq 0$ or $\dot{U}(C) \geq 0$ to the statement of the maximization problem given by (5), where the pure rate of time preference, ρ, is assumed to be positive. An equivalent condition, shown by Pezzey (1994), is that the

value of total cpital be non-declining. In terms of our basic model this can be written as

$p\dot{K} + q\dot{X} \geq 0$.

As described by Toman et al. (1995), the opsustimal path typically has two phases. In the initial phase, consumption and utility both rise and total investment is positive ($p\dot{K} + q\dot{X} > 0$); that is, the accumulation of produced capital more than offsets the depletion of natural capital. In the second phase, investment in man-made capital is equal to resource rent (Hartwick's condition: $p\dot{K} + q\dot{X} = 0$) and consumption remains constant. Advocates of opsustimal growth view it as both just and superior to maximin welfare. It permits an initial period of capital accumulation, so that countries are not held hostage to their initial endowments. As a result, the level of constant consumption in the second phase of the opsustimal path dominates maximin consumption. But, as noted by Toman et. al. (1995), the opsustimal approach does not resolve how the social welfare function should directly reflect concerns about intergenerational equity. To proponents of utilitarian welfare, the imposition of non-declining consumption or utility as a side constraint to achieve sustainability and intergenerational equity is without foundation.

IV. GOLDEN RULES AND SUSTAINABLE DEVELOPMENT

We propose an approach to sustainability that retains the utilitarian framework but avoids ad hoc side constraints or the restrictive assumptions needed to guarantee the existence of an optimum consumption trajectory. This approach is based on a golden rule of resource management and capital accumulation obtained by setting the rate of time preference, ρ, equal to zero. This form of golden rule serves as an alternative to the "green golden rule" developed by Beltratti, Chichilnisky and Heal (1993), which is based on the notion of sustainable preferences.

The problem of discounting has received considerable attention in the natural resource literature, but controversy remains. Ramsey (1928) is often cited for his forceful pronouncement that discounting is "ethically indefensible." This view has been challenged on technical, empirical and theoretical grounds. The technical difficulty with a utility discount rate of zero is discussed for example by Heal (1993) and is sometimes referred to as the "cake eating problem." In a simple cake eating model of a finite resource, Heal (1993) shows that the implication of zero utility discounting is zero consumption over an infinite time horizon. We argue below that the existence of a backstop technology or a steady state in the case of a renewable resource solves this technical problem.

Olson and Bailey (1981) reject zero discounting on empirical grounds. Using a partial equilibrium model with an exogenous positive interest rate, Olson and Bailey argue that an individual rate of pure time preference that is zero would impoverish the present. People would cut current consumption down to the subsistence level to provide for future generations. Since people in fact do not choose subsistence in the present, the individual rate of pure time preference must be positive. In our view, this argument does not close the case; the pure rate of time preference should be studied in a general equilibrium framework, where the interest rate is endogenous. Moreover, an individual rate of pure time preference that is positive does not necessarily prevent a social planner from applying neutral weighting across generations as a matter of normative policy.

Koopmans (1965) discusses the theoretical problem associated with zero discounting, but then identifies a practical solution. Citing previous work by Koopmans (1960) and Koopmans, Diamond and Wlliamson (1964), Koopmans states that ".... there does not exist a utility function of all consumption paths, which at the same time exhibits timing neutrality, and satisfies other reasonable postulates which all utility functions used so far have agreed with." The solution to the problem builds on the early work of Ramsey (1928). The idea is to identify a subset of all feasible consumption paths on which one can define a neutral utility function. A member of this subset is characterized as an eligible path, and Ramsey's criterion for eligibility is approach of the path to a " bliss point". Koopmans adapts this method: "We shall find that in the present case of a steady population growth, the golden rule path can take the place of Ramsey's state of bliss in defining eligibility." Using the golden rule path, Koopmans (1965) demonstrates that each eligible path is superior to each path that is ineligible. Moreover, one can rank eligible paths and determine one that is optimal. The results of Section V of our present paper show the possibility of golden rule states for an economy with essential natural resources, thereby permitting use of the Ramsey-Koopman technique to apply neutral weighting across generations.

In further support of neutral generational weighting, Burton (1993) makes the argument that the standard approach to utility discounting confuses two key but distinct concepts: the discount rates that reflect the intertemporal preferences of members of society and issues of intergenerational equity. Burton incorporates both considerations into models of optimal resource harvesting by postulating two separate discount rates:

a personal discount rate, designated by β, which reflects the rate of pure time preference of individuals; and a generational discount rate, ρ, which addresses society's degree of concern for intergenerational equity. Endress (1994) extends this analysis to the case of an overlapping generations model of a production economy with a renewable resource. The results of this model show that the optimum trajectory is governed at each moment in time by relationships among aggregate quantities and the generational discount rate, ρ, but not the personal discount rate, β.

We believe that this finding has important implications for modeling economic growth in a manner compatible with intergenerational equity. Stewardship for the future can be accommodated by setting the generational discount rate, ρ, equal to zero. Such an approach to intergenerational equity would not conflict with the possible existence of a positive personal discount rate, β, governing intertemporal preferences over the lifetime of the individual.

In deriving golden rule conditions, we distinguish three types of steady state according to the taxonomy presented in Roumasset and Wang (1992). We assume in each case that the economy enters the steady state at some endogenous time T.

(1) Economically renewable resources. This is thc case in which the steady state stock is positive and the resource extraction cost, θ, is less than the cost of the backstop; that is, $X(T) > O$ and $\theta(X(T)) < \theta_{\mathbf{b}}$. Equations (13) and (14) represent the modified golden rule for this situation. By setting $\rho = 0$, we obtain the golden rule for an economically renewable resource:

$$F_K = \delta \qquad F_R = \theta - \frac{\theta^{\prime}(X)G(X)}{G^{\prime}(X)} \qquad (17)$$

(2) Exhaustible resources. The case of non-renewable resources is covered by setting $G(X) = 0$ in equation (13). More generally, however, we consider the possibility that renewable resources might be extinctable along the optimum trajectory. In this type of steady state, $X(T) = 0$ and $\theta(T) = \theta_{\mathbf{b}}$. Therefore, for the case of exhaustible resources, the golden rule becomes

$$F_k = \delta \tag{18}$$
$$F_R = \theta_{\mathbf{b}}$$

(3) Replaceable resources. In this case, the backstop price, $\theta_{\mathbf{b}}$, becomes binding before the stock level, X, is allowed to reach the golden rule level, or before it is exhausted. For this situation, $X(T) > 0$, but $\theta(X(T)) = \theta_{\mathbf{b}}$. For the case of replaceable resources, the golden rule is again represented by conditions (18).

In Figure 1, we sketch plausible trajectories of per capita consumption representing modified golden rule growth paths. These trajectories are analogous to those depicted in Diagram 10.3 of Dasgupta and Heal (1979), with the addition of a backstop substitute. For the case of a Cobb-Douglas production function, Dasgupta and Heal (1974, 1979) showed that the consumption trajectory, for the case of a non-renewable resource, will have at most one peak. Moreover, the lower the rate of time preference, the further in the future will be the peak. Trajectory 2 of Figure 1 eventually dominates maximin consumption, while trajectory 1 does not.

We may also compare the golden rule to the maximin rule as alternative standards of intergenerational equity. One plausible scenario is illustrated in Figure 2. By definition, the maximin path yields the maximum possible level of constant per capita consumption less than or equal to that rendered by a golden rule steady state. As the figure shows, this may result in large and sustained (and therefore infinite) losses in the future in order to raise consumption in the present and near future by small increments.

V. SUSTAINABLE GROWTH AND GOLDEN RULE NET NATIONAL PRODUCT

Consideration of the golden rule helps clarify the connection between net national product and sustainable growth. In their discussions of sustainability, Mäler (1991) and Johansson (1993) the profile of net national product. If the maximized current value Hamiltonian is

$$\bar{H}^* = U(C) + \bar{\lambda}^*\dot{K}^* + \bar{\psi}^*\dot{X}^*, \tag{19}$$

it is a simple exercise to show that

$$\frac{d\bar{H}^*}{dt} = \rho[\bar{\lambda}^*\dot{K}^* + \bar{\psi}^*\dot{X}^*] \tag{20}$$

Mäler (1991) then offers a definition of sustainability based on the time profile of the current value Hamiltonian; specifically, sustainable growth, according to Mäler, is growth for which $d\bar{H}/dt \geq 0$. For $\rho > 0$, this condition implies that

$$[\bar{\lambda}^*\dot{K}^* + \bar{\psi}^*\dot{X}^*] \geq 0 \tag{21}$$

which says that the value of the capital stock, measured in current year prices, never decreases. A special case is that for which $d\bar{H}/dt = 0$. Then the associated condition on capital is

$$[\bar{\lambda}^*\dot{K}^* + \bar{\psi}^*\dot{X}^*] = 0 \tag{22}$$

This is Harwick's rule, which keeps net investment equal to zero; all resource rents are invested in capital accumulation, but no additional saving is pursued.

Pezzey (1994) evaluates this as well as other possible sustainability constraints on aggregate growth. An alternative, and perhaps more direct approach to sustainability is based on a comparative evaluation of consumption trajectories. We suggest the concept of ***relative sustainability*** as characterizing economic growth that supports a consumption trajectory eventually meeting or exceeding maximin consumption or some other floor level of consumption chosen by the planner. This approach to sustainable growth is most readily addressed by returning to the golden rule for resource management and capital accumulation.

As a point of departure, consider utility maximization without time discounting, along the lines of Koopmans (1965). Solving this problem requires that we incorporate a Ramsey type "bliss" point into the integrand to allow convergence of the integral. In the context of our basic model, the golden rule steady state level of consumption $\hat{C}$ is given by

$$\hat{C} = F(\hat{K},\hat{R}) - \delta\hat{K} - \theta\hat{R} \tag{23}$$

where $\hat{K}$ and $\hat{R}$ are determined by the golden rule condition for F_K and F_R. As discussed earlier, these conditions depend on whether the resource is economically renewable, exhaustible or replaceable. The bliss point for the economy in terms of aggregate utility is then $U(\hat{C})$. The Ramsey problem with respect to our basic model becomes

$$\begin{aligned} &\text{Max}\int_0^{\infty} [U(C) - U(\hat{C})]dt \\ \text{s.t.}\quad &\dot{K} = F(K,R) - \delta K - \theta R - C, \quad K(0) = K_0 \\ &\dot{X} = G(X) - R, \quad X(O) = X_O \end{aligned} \tag{24}$$

The Hamiltonian for this problem is

$$H = [U(C) - U(\hat{C})] - \lambda[F(K,R) - \delta K - \theta R - C] + \psi[G(X) - R] \tag{25}$$

Application of the maximum principle yields the familiar Ramsey condition and generalized Hotelling Rules for the transition to the steady state, but now with $\rho = 0$:

$$\eta(C)\frac{\dot{C}}{C} = (F_K - \delta) \tag{26}$$

$$(F_R - \theta) = \frac{1}{(F_K - \delta)}\{\dot{F}_R - (F_R - \theta)G'(X) - \theta'(X)G(X)\} \tag{27}$$

Of special interest, however, is the Hamiltonian. Since the Ramsey problem can be classified as autonomous with no time discounting, the Hamiltonian along optimum trajectory remains constant; that is, $dH^*/dt = 0$, where

$$H^* = [U(C^*) - U(\hat{C})] + \lambda^*\dot{K}^* + \psi^*\dot{C}^* \tag{28}$$

Now in the steady state, $\dot{K} = \dot{X} = 0$ and $C^* = \hat{C}$. This implies that $\hat{H} = 0$ and, consequently, $H = 0$ for all time t. Rearranging the expression for H gives

$$U(\hat{C}) = U(C^*) + \lambda^*\dot{K}^* + \psi^*\dot{X}^* \tag{29}$$

Bliss point utility, $U(\hat{C})$, therefore serves as the golden rule net national product. It remains constant over time; as C^* increases toward $\hat{C}$, $\dot{K}^*$ and $\dot{X}^*$ approach zero, and λ^* and ψ^* decrease monotonically. We illustrate the situation in Figure 3, which is based on the Weitzman (1976). This aids graphical representation of the dynamic change in the economy over time.

Curve (aa) of Figure 3 represents the feasibility frontier of the economy at time $t = 0$. Consumption level, $\bar{C}$, is the maximum attainable level of consumption at time $t = 0$ if no investment were to take place and $U(\bar{C})$ is the associated level of utility. The utility-investment pair $(U(C^*), K^*)$ lies on the optimal trajectory to the steady state. With positive investment, capital is accumulated, and the feasibility frontier moves outward and upward and toward the right. Because $U(C)$, rather than C is plotted on the vertical axis, equal increments in maximum attainable consumption starting from $\bar{C}$ are reflected as diminishing increments in maximum attainable utility. Therefore, movement of the feasibility frontier is not symmetric. As the frontier moves outward, consumption and utility of consumption increase over time. The shadow prices of capital, λ^*, is represented at each time t, by the slope of the line tangent to the prevailing feasibility frontier and passing through $U(\hat{C})$ on the vertical axis. As the economy evolves, the feasibility frontier advances until maximum attainable consumption reaches the golden rule level, $\hat{C}$, and $\dot{K}^* = 0$, as depicted by curve (bb).

We introduce an important caveat in the interpretation of golden rule net national product. When expressed in terms of utility of consumption, golden rule net national product is constant over time. However, when measured relative to the aggregate consumption numeraire at each time t, golden rule net national product actually increases. Dividing NNP by $U'(C)$ yields $U(C)/U'(C)$; while $U(C)$ is constant, $U'(C)$ decreases as C^* approaches $\hat{C}$ along the optimum trajectory to the golden rule steady state.

Throughout this study, we have employed the services of the family of utility functions $U(C) = - C^{-(\eta-1)}$, $\eta > 1$, exhibiting constant elasticity of marginal utility. Dasgupta and Heal (1979) observe that the parameter, η, reflects the degree to which society is egalitarian in the distribution of consumption across generations (alternatively, η may be interpreted as a measure of relative risk aversion). As h gets larger and larger, the initial level of consumption increases and the consumption trajectory becomes flatter. This is illustrated in Figure 4. Note, however, that there is an upper limit to the initial level of aggregate consumption. Compatible with the feasibility set underlying the economy at t = 0, this level is $\bar{C}$ in Figure 4, the level of maximum attainable consumption. With infinite elasticity of marginal utility, society chooses the point $(U(\bar{C}),O)$ on the feasibility frontier of Figure 3 at time t = 0. At this point net investment is zero, implying Hartwick's rule for the case of an economy with both man-made capital and ecological capital.

How does maximin relate to thc golden rule steady state and thc bliss point of the economy? The possibility of capital accumulation guarantees that $\bar{C} < \hat{C}$. However, for the case of infinite elasticity of marginal utility, $U(\bar{C}) = U(\hat{C})$; the utility of maximum attainable consumption at time t = 0, now becomes the economy's bliss point. This observation, however, does not establish maximin welfare as the correct approach to intergenerational equity. Imposition of maximin in welfare on a society whose preferences could be represented by an aggregate utility function with $\eta < \infty$, would constitute a case of economic distortion. But it would represent a major distortion of justice as well, by robbing future generations of the golden rule standard of living.

The other extreme is represented by the case, $U(C) = C$, for which the elasticity of marginal utility is zero. The Ramsey problem now becomes

$$\mathrm{Max} \int_0^\infty [C - \hat{C}]dt \qquad (30)$$

$$\dot{K} = F(K,R) - \delta K - \theta R - C, \qquad K(0) = K_0$$

$$\dot{X} = G(X) - R, \qquad X(0) = X_0$$

Geometrically, this is equivalent to choosing the feasible consumption trajectory, $C(t)$, that minimizes the area between the horizontal line $\hat{C}$ and the trajectory $C(t)$. To minimize this area, the economy should accumulate capital as quickly as possible up to the point that $F_K = \delta$ to permit the most rapid rise of consumption toward the golden rule steady state. Such a program of capital accumulation might entail totally impoverishing the present, so that output at time $t = 0$, net of capital depreciation and extraction costs, is fully allocated to investment.

The contrast between the two extremes is striking. For the case $\rho = 0$ and $\eta \to \infty$, the present gains at the expense of future generations, who forgo the opportunity to enjoy the golden rule steady state level of consumption, $\hat{C}$. At the other extreme, we have $\rho = 0$ and $\eta = 0$. In this case, future generations reap the benefits of golden rule consumption owing to the profound sacrifice of the present generation.

VII. CONCLUDING REMARKS

Drawing on the field of neoclassical growth theory, this paper has contributed toward putting the concept of sustainable growth on a firmer theoretical foundation. Under relatively general conditions, we derived golden rules as a basis of optimal capital accumulation and natural resource management in growing economies. This approach is more compatible with sustainable growth than trajectories based on constrained utilitarian optimization .

There are several drawbacks to the "opsustimal" (constrained utilitarian optimization) paradigm. In general, constrained optimization models are likely to be in conflict with generally accepted axioms of rational choice. That turns out to be the case here.

First, constrained optimization cannot provide a full ranking of alternatives because alternatives that violate the constraint cannot be compared. In the case of opsustimal growth with a finite stock of non-renewable resources, if either the elasticity of substitution between natural capital and produced capital is less than one or the output elasticity of natural capital is greater than that of produced capital (with elasticity of substitution less than or equal to one) then the sustainability constraint renders the opsustimal problem infeasible. Clearly, among all the infeasible consumption trajectories some trajectories are preferable to others. Constrained welfare maximization does not provide such a ranking.

Second, an opsustimal path may be strictly dominated by an unconstrained welfare maximum. Recall that the typical profile of an optimal consumption path with a positive but modest utility discount rate and either a backstop or renewable resource available is single-peaked and then asymptotically approaches some positive lower bound consumption level. If that lower bound is above the maximum consumption level for opsustimal growth, then the latter is strictly dominated under the axiom that more is better than less.

Finally, recall that the optimal consumption profile is strictly increasing if the utility discount rate is zero. In that case the sustainability is redundant.

Ultimately, the elements of sustainable growth are efficient use of thc economy's resources and stewardship for the future. Sustainability is best achieved not by the imposition of artificial constraints on economic growth, but by enlightened policies based on golden rules of capital accumulation and resource management. This approach to sustainability rests on the firm theoretical foundations established by true pioneers of economic dynamics and growth: Frank Ramsey, Harold Hotelling, and Tjalling Koopmans.

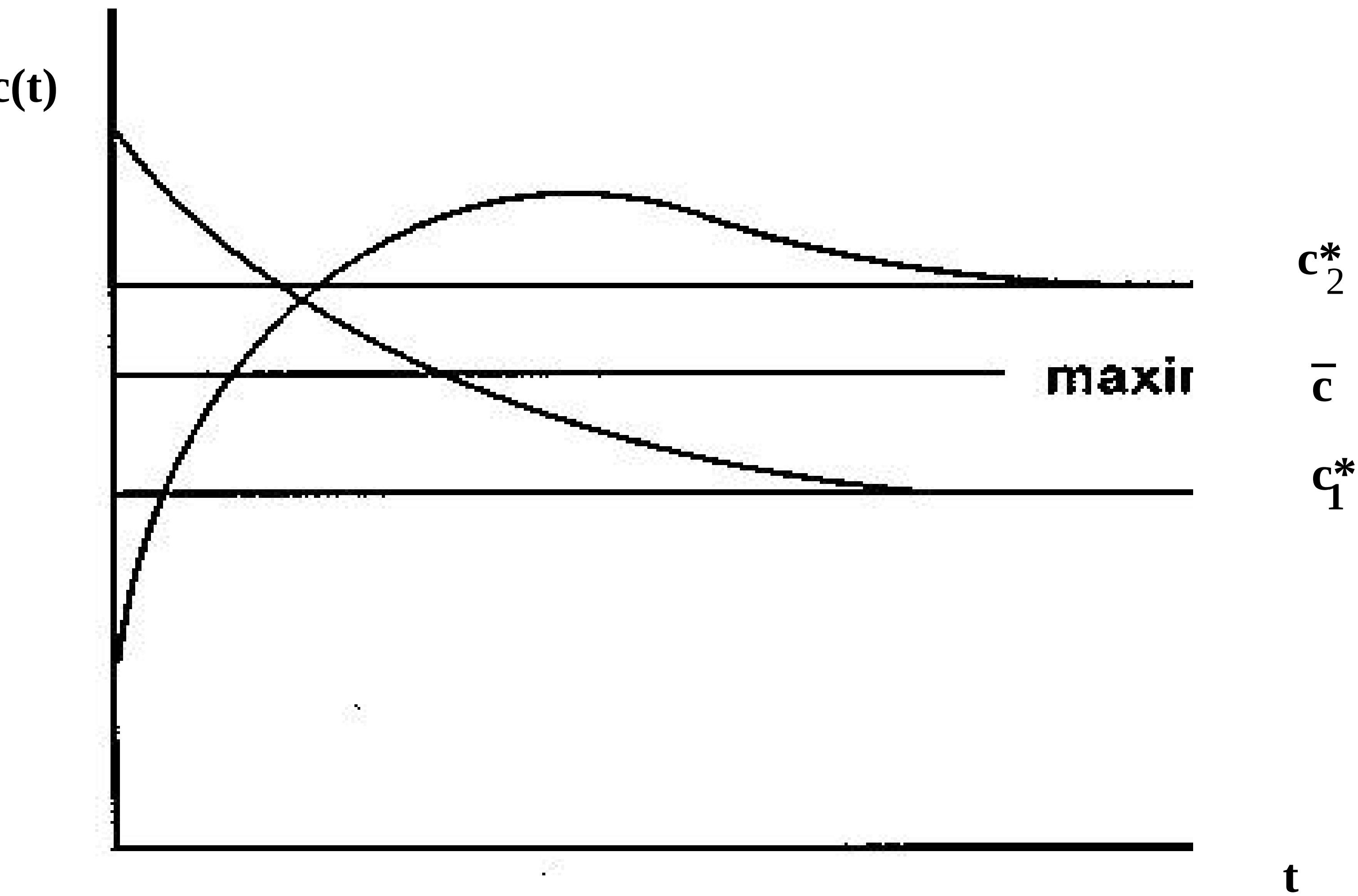

Figure 1

Plausible Consumption Trajectories Leading to the Modified Golden Rule

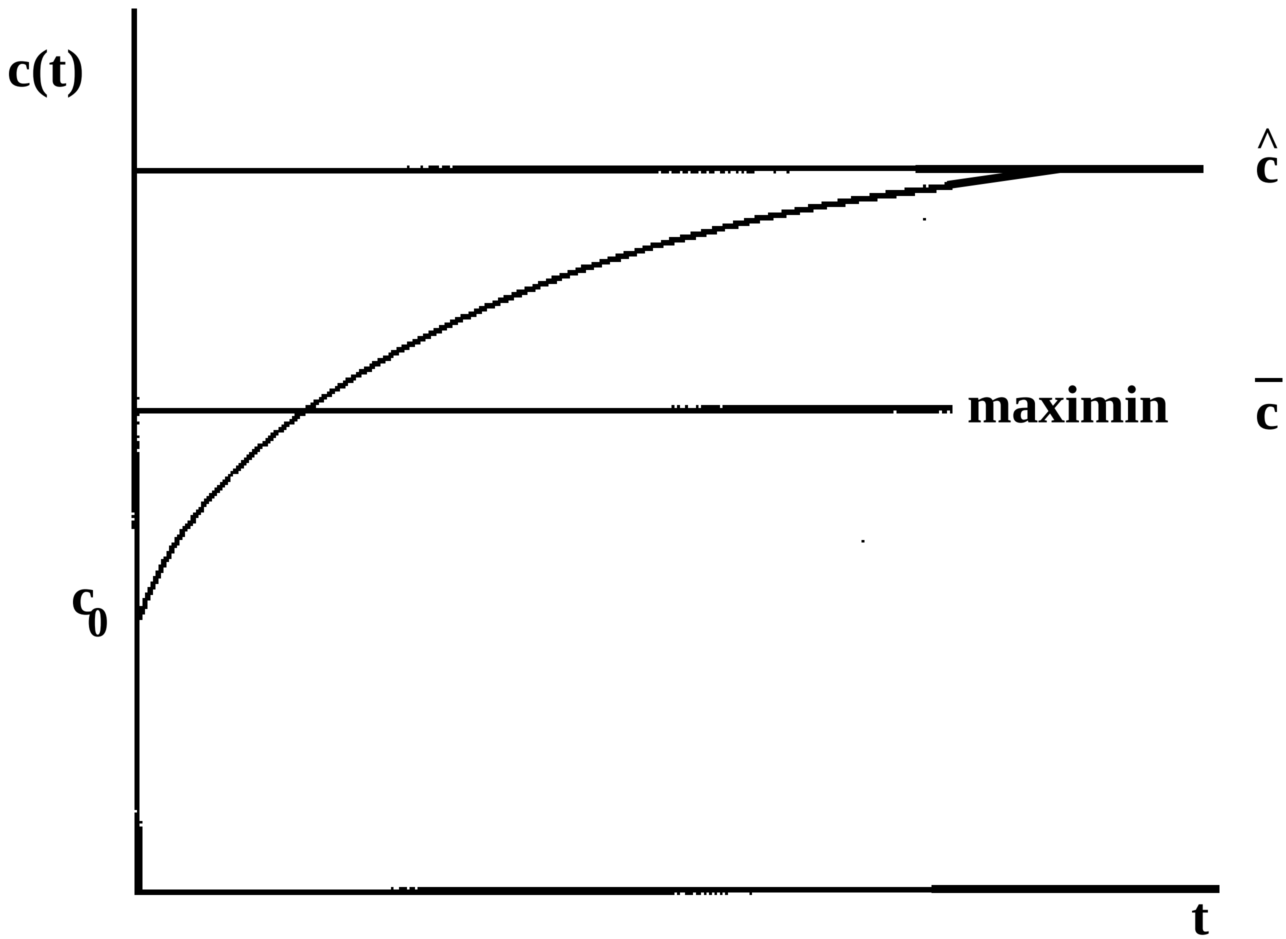

Figure 2

Golden Rule Turnpike vs. Maximin Path As Alternative Trajectories of Sustainable Consumption

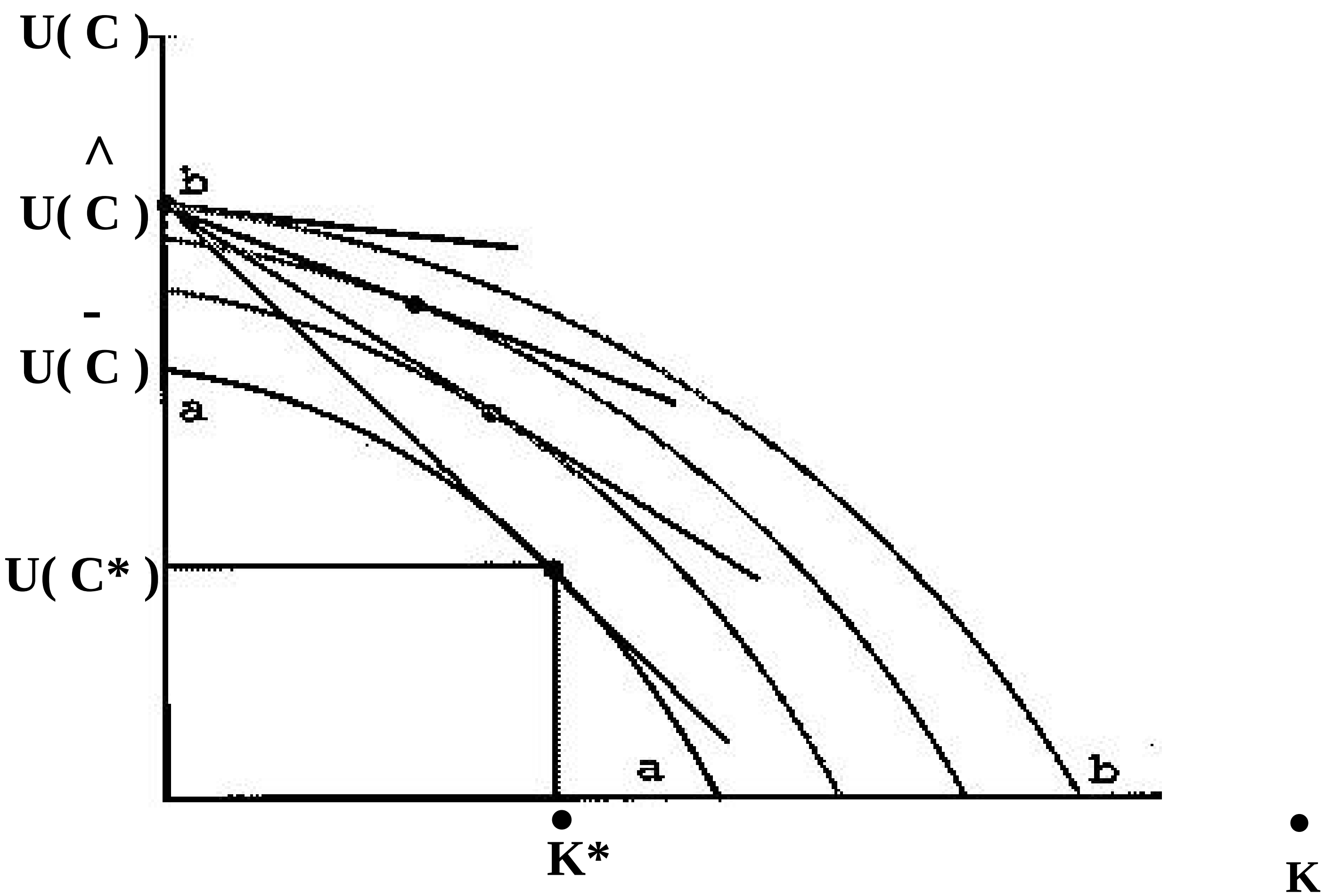

Figure 3

Golden Rule NNP

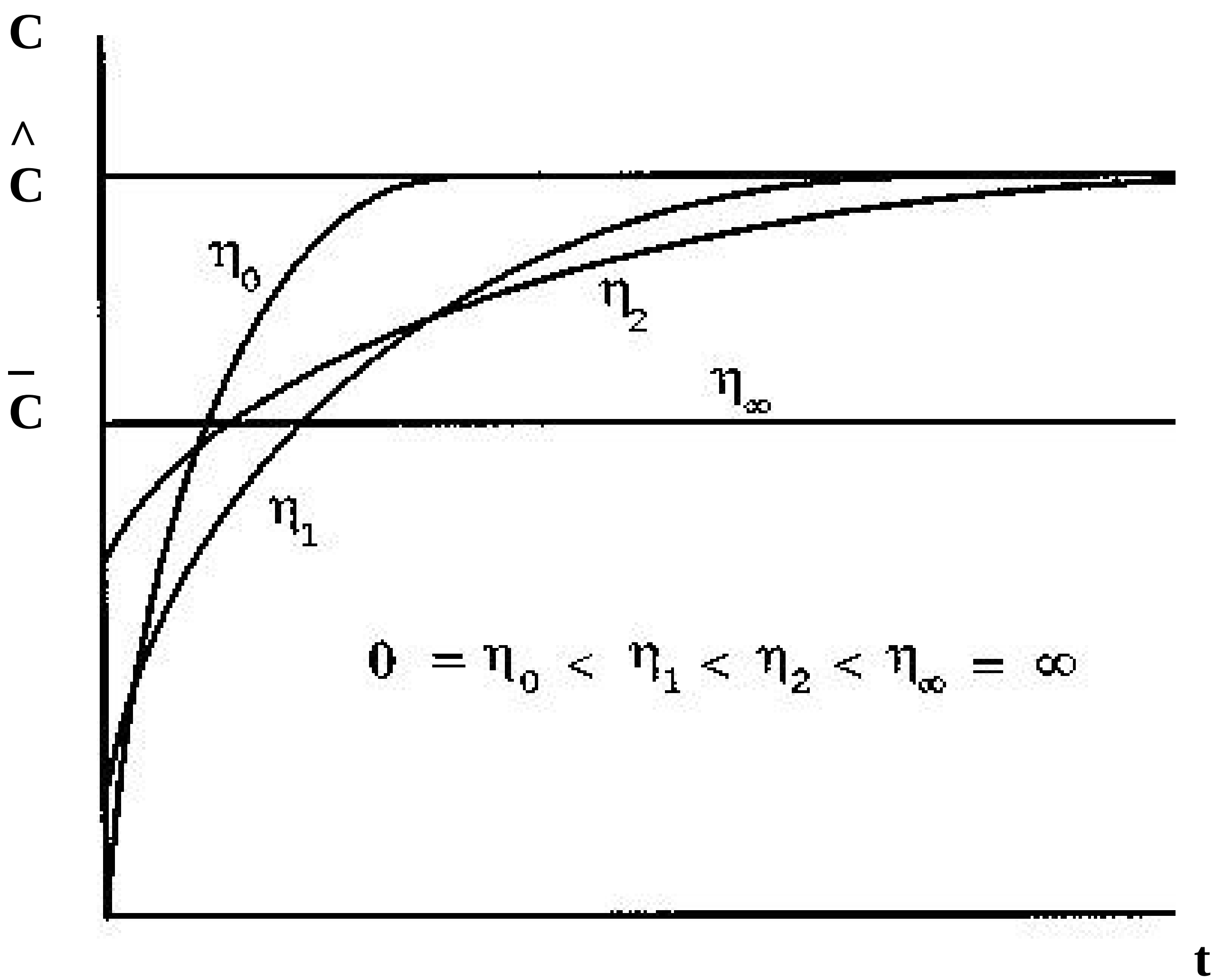

Figure 4

Effect of Elasticity of Marginal Utility On Golden Rule Trajectories

APPENDIX

The planners problem is to maximize utilitarian welfare subject to dynamic constraints on capital accumulation and resource use:

$$\text{Max} \int_{o}^{\infty} U(C)e^{-\rho t}\, dt$$

$$\text{s.t.} \quad \dot{K} = F(K,R) - \delta K - \theta(X)R - C$$

$$\dot{X} = G(X) - R$$

The Hamiltonian expression is

$$H = U(C)e^{-\rho t} + \lambda[F(R,K) - \delta K - \theta(X)R - C] + \psi[G(X) - R(t)]$$

The standard necessary conditions for this optimal control problem are:

$$\frac{\partial H}{\partial K} = U^{\prime}(C)e^{-\rho t} - \lambda = 0$$

$$\frac{\partial H}{\partial R} = \lambda[F_R - \theta] - \psi = 0$$

$$\dot{\lambda} = -\frac{\partial H}{\partial K} = -\lambda[F_K - \delta]$$

$$\dot{\psi} = -\frac{\partial H}{\partial X} = \lambda[\theta^{\prime}(X)R] - \psi G^{\prime}(X)$$

From the first and third conditions,

$$\dot{\lambda} = U^{\prime}(C)(-\rho)e^{-\rho t} + \dot{U}^{\prime}(C)e^{-\rho t}$$

and

$$\dot{\lambda} = -U^{\prime}(C)e^{-\rho t}[F_K - \delta]$$

Equating expressions for λ and rearranging yields

$$-\frac{\dot{U}'(C)}{U'(C)} = [F_K - (\delta + \rho)]$$

or $\eta \frac{\dot{C}}{C} = F_K - (\delta + \rho)$ where $\eta = \frac{-U''(C)C}{U'(C)}$

From the second necessary condition:

$$\dot{\psi} = \lambda[\dot{F}_R - \theta'(X)\dot{X}] + \dot{\lambda}(F_R - \theta)$$

$$= \lambda[\dot{F}_R - \theta'(X)(G(X) - R)] - \lambda(F_K - \delta)(F_R - \theta)$$

But from the fourth necessary condition,

$$\dot{\psi} = \lambda[\theta'(X)R] - \psi[G(X)]$$

$$= \lambda[\theta'(X)R] - \lambda[F_R - \theta]G'(X)$$

Equating expressions for y and rearranging yields

$$[\dot{F}_R - \theta'(X)G(X)] - (F_K - \delta)(F_R - \theta) = -(F_R - \theta)G'(X)$$

or $(F_R - \theta) = \frac{1}{(F_K - \delta)}\{\dot{F}_R + (F_R - \theta)G'(X) - \theta'(X)G(X)\}$

BIBLIOGRAPHY

Asheim, G.B., 1988, "Rawlsian Intergenerational Justice as a Markov-perfect Equilibrium in A Resource Technology," ***Review of Economic Studies*** 55 (3), 469-84.

Beltratti, Andrea, Graciela Chichilnisky and Geoffrey Heal, (1993). "Sustainable Growth and the Green Golden Rule," in I. Golden and L.A. Winters***, The Economics of Sustainable Development,*** Cambridge University Press***.***

Burton, P., 1993, "Intertemporal Preferences and Intergenerational Equity Considerations in Optimal Resource Harvesting," ***Journal of Environmental Economics and Management***, 24.

Chiang, A.C, 1992, ***Elements of Dynamic Optimization***, McGraw Hill.

Clark, C.W., 1991, ***Mathematical Bioeconomics***, Wiley Interscience.

Dasgupta, P., 1974, "On Some Alternative Criteria for Justice Between Generations," ***Journal of Public Economics***, 3: 405-423.

Dasgupta, P.S. and G.M. Heal, 1974, "The Optimal Depletion Exhaustible Resources". ***Review of Economic Studies***, Symposium Issue (December): 3-28.

Dasgupta, P.S. and G.M. Heal, 1979, Economic Theory and Exhaustible Resources, Cambridge University of Press.

Dasgupta, P.S. and K.G. Mäler, 1991, "The Environment and Emerging Development Issues." Development Economics Research Program, DEP No. 28.

Diamond, P., and J. Mirrlees, 1976, "Private Constant Returns and Public Shadow Prices," ***The Review of Economic Studies***, 43.

Diewert, W.E., 1983, "Cost-Benefit Analysis and Project Evaluation: A Comparison of Alternative Approaches," ***Journal of Public Economics***, 22.

Dixit, A., R. Hammond, and M. Hoel, 1980, "On Hartwick's Rule for Regular Maximin Paths of Capital Accumulation and Resource Depletion," ***Review of Economic Studies***, 45.

Endress, L.H. and J.A. Roumasset, 1994, "Golden Rules for Sustainable Resource Management," ***The Economic Record***, Vol. 70, No. 210, 267-277.

Hartwick, J.M., 1977, "Intergenerational Equity and the Investing of Rents from Exhaustible Resources." ***American Economic Review***. 67 (Dec): 972-4

Hartwick, J.M., 1978, "Investing Returns from Depleting Renewable Resource Stocks and Intergenerational Equity." ***Economics Letters***, 1: 85-8.

Heal, G.M., 1976, "The Relationship Between Pride and Extraction Cost for a Resource with a Backstop Technology." ***Bell Journal of Economics***, 7: 371-8.

Heal, G.M., 1993, "Optimal Use of Exhaustible Resources," in A.V. Kneese and J.L. Sweeney, ***Handbook of Natural Resource and Energy Economics***, Vol. III, North Holland.

Johansson, P., 1993, ***Cost-Benefit Analysis of Environmental Change***, Cambridge University Press.

Koopmans, T.C., 1960, "Stationary Ordinal Utility and Impatience," ***Econometrica***, April 1960, 287-309.

Koopmans, T.C., P.A. Diamond and R.E. Williamson, "Stationary Utility and Time Perspective," ***Econometrica***, January-April 1964, 82-100.

Koopmans, T.C., 1965, "On the Concept of Optimal Growth***." The Economic Approach to Development Planning***. Chicago: Rand McNally.

Lewis, T.R., 1982, "Sufficient Conditions for Extracting Least Cost Resource First." ***Econometrica***, 50: 1081-3.

Maler, K.G., 1991, "National Accounts and Environmental Resources***," Environmental and Resource Economics***," 1: 1-15.

Olson, M. and Bailey, M.J., 1981, "Positive Time Preference***," Journal of Political Economy***, Vol. 89, No.1, 1-25.

Panayotou, T., 1993, ***Green Markets: The Economics of Sustainable Development***, Institute for Contemporary Studies, San Francisco, California.

Pearce, D., E. Barbier and A. Markandya, 1990***, Sustainable Development: Economics and Environment i***n the Third World, Edward Elger.

Pearce, D.W. and J.J. Warford, 1993, ***World Without End: Economics, Environment and Sustainable Development***, Published for the World Bank by Oxford University Press.

Pezzey, J., 1989, "Economic Analysis of Sustainable Growth and Sustainable Development," Environmental department Working Paper No. 15, World Bank.

Pezzey, J., 1994, "The Optimal Sustainable Depletion of Non-Renewable Resources," Presented at the 1994 AERE Workshop on Integrating the Environment and the Economy, Boulder Colorado, June 5-6.

Ramsey, F.P., 1928, "A Mathematical Theory of Saving." ***Economic Journal***. 138: 543-59.

Rawls, J., 1971, ***A Theory of Justice***, Harvard University Press.

Roumasset, J. and D. Wang, 1992, "Optimal Depletion of Technically Renewable Resources," Working paper, Economics Department, University of Hawaii.

Sala-i-Martin, X., 1990, Lecture Notes on Economic Growth (1): Introduction to the Literature and Neoclassical Models. NBER Working Paper No. 3563.

Siebert, H., 1992, ***Economics of the Environment***, Theory and Policy, 3rd Ed., Springer Verlag, Berlin/New York.

Solow, R.M., 1986, "On the Intergenerational Allocation of Natural Resources," ***J. of Economics***, 88 (1).

Solow, Robert, 1992, ***An Almost Practical Step Toward Sustainability***. Washington, D.C., Resources for the Future.

Solow, R.M., 1993, "Sustainability, An Economists Perspective," in Dorfman and Dorfman, ***Economics of the Environment***, Third Edition, Norton.

Toman, M.A., J. Pezzey, and J. Krautkraemer, 1995, "Neoclassical Economic Growth Theory and Sustainability," in D.W. Bromley, ***The Handbook of Environmental Economics,*** Blackwell.

Weitzman, M.L., 1976, "On the Welfare Significance of National Product in a Dynamic Economy," ***Quarterly Journal of Economics***, 90.